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# PID Controller Tuning Rules for Robust step response of First-Order-Plus-Dead-Time models

Ramon Vilanova

**Abstract**—This communication addresses the problem of tuning a PID controller for step response. The tuning is based upon a First Order Plus Time Delay (FOPTD) model and aims to achieve a step response specification while taking into account Robustness considerations. The industrial ISA-PID formulation is chosen. A tuning rule is derived first where the four parameters of the ISA-PID are determined by means of two new parameters: one parameter is related to the desired closed-loop time constant and the other one to the robustness level. On a second step these two parameters are set to a fixed value in order to get a simple and automatic rule that directly gives the controller parameters in terms of the process model parameters. The proposed automatic tuning rule is compared with other known tunings.

**Index Terms**—PID Control, Robustness, FOPDT Models.

## I. INTRODUCTION

Proportional-Integrative-Derivative (PID) controllers are with no doubt the most extensive option that can be found on industrial control applications. Their success is mainly due to its simple structure and meaning of the corresponding three parameters. This fact makes PID control easier to understand by the control engineer than other most advanced control techniques.

Because of the widespread use of PID controllers it is interesting to have simple but efficient methods for tuning the controller. In fact, since Ziegler-Nichols proposed their first tuning rules [1], an intensive research has been done. From modifications of the original tuning rules [2], [3], [4] to a variety of new techniques: analytical tuning [5], [6]; optimization methods [7], [8]; gain and phase margin optimization [7], [9], just to mention a few.

Recently, tuning methods based on optimization approaches with the aim of ensuring good stability robustness have received attention in the literature [10], [11]. Also, great advances on optimal methods based on stabilizing PID solutions have been achieved [12], [13]. However these methods, although effective, use to rely on somewhat complex optimization procedures and do not provide tuning rules. Instead, the tuning of the controller is defined as the solution of the optimization problem.

The purpose of this paper is to obtain PID tuning rules based on a combination of a simple model description;

First Order plus Time Delay (FOPTD); and closed loop specifications with robustness considerations. The tuning rules are given in two forms: firstly parameterized in terms of desired time constant and robustness level and, secondly, a completely automatic tuning determined by the process parameters. To get the results as close as possible to the industrial situation, the widely used ISA structure [7] is chosen for the PID control law.

The paper is organized as follows. Section 2 presents the problem formulation: process model, PID structure and the optimization problem based on a min-max formulation. Section 3 solves the min-max optimization problem and provides the controller structure and expression for the PID parameters. Starting from the optimal controller structure, Section 4 introduces the Robust stability constraint and provides the guideline for a Robust Tuning. Starting from the previous guidelines a simple tuning rule for automatic tuning is provided in Section 5. Simulation examples are presented in section 6 where the automatic tuning is compared with other known tuning rules. Finally, in section 7, conclusions and considerations for further extensions are conducted.

## II. PROBLEM FORMULATION

In this section the controller equations are presented as well as the assumed process model structure and the optimization problem that is posed in order to tune the PID controller.

### A. PID Controller

There exists different ways to express the PID control law [14]. In this paper we concentrate on the ISA PID control law [7]

$$u(s) = K_p \left[ br(s) - y(s) + \frac{1}{sT_i}(r(s) - y(s)) + \frac{sT_d}{1 + sT_d/N}(cr(s) - y(s)) \right] \quad (1)$$

where  $r(s)$ ,  $y(s)$  and  $u(s)$  are the Laplace transforms of the reference, process output and control signal respectively.  $K_p$  is the PID gain,  $T_i$  and  $T_d$  are the integral and derivative time constants, finally  $N$  is the ratio between  $T_d$  and the time constant of an additional pole introduced to assure the properness of the controller. Parameters  $b$  and  $c$  are called

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set-point weights and constitute a simple way to obtain a 2-DOF controller. As their choice does not affect the feedback properties of the resulting controlled system, with no loss of generality here we will assume  $b = c = 1$ . This way, the PID transfer function we will work with can be written as

$$K(s) = K_p \frac{1 + s(T_i + \frac{T_d}{N}) + s^2 \frac{T_i T_d (N+1)}{N}}{s T_i (1 + s \frac{T_d}{N})} \quad (2)$$

### B. Process Model

An important category of industrial processes can be represented by a First Order Plus Dead Time (FOPDT) model as.

$$G_n(s) = \frac{K e^{-Ls}}{1 + Ts} \quad (3)$$

where  $K$  is the process gain,  $T$  the time constant and  $L$  the time delay. This kind of models are easy to determine by means of a simple step response experiment to get the process reaction curve. In order to deal with the delay term is usual to use a rational approximation. Here we will work with the following simple first order Taylor expansion of the  $e^{-Ls}$  term

$$e^{-Ls} \approx 1 - Ls. \quad (4)$$

### C. Optimization problem

In order to take into account robustness considerations, the design problem must be posed accordingly. One, rather usual, approach is to use frequency dependent uncertainty descriptions and to include them into the design problem [15]. Suppose the real process  $G(s)$  is given by the nominal model (3). An uncertainty description based on a multiplicative model error,  $\Delta_m(s)$  is defined as

$$\Delta_m(s) \doteq \frac{G(s) - G_n(s)}{G_n(s)} \quad (5)$$

It is well known that a controller,  $K(s)$ , that stabilizes the control system on the nominal system, also stabilizes all the control systems built up around a family  $\mathcal{F}$  of plants such that

$$\|W_m(s)T(s)\|_\infty < 1 \quad (6)$$

where  $T(s)$  is the nominal Complementary Sensitivity transfer function:

$$T(s) = \frac{K(s)G_n(s)}{1 + G_n(s)K(s)} \quad (7)$$

and  $W(s)$  is a frequency dependent weight that defines the family of plants:

$$\mathcal{F} = \{G(s) = G_n(s)(1 + \Delta_m(s)) : |\Delta_m(jw)| < |W_m(jw)|\} \quad (8)$$

However if one uses the Internal Model Control paradigm (IMC) that can be found in [15]; and exemplified in fig

(1); it turns out that  $T(s)$  has a very simple expression;  $T(s) = G_n(s)C(s)$ ; in terms of the so called IMC controller, or Youla parameter, for stable plants.

The IMC synthesis gets  $C(s)$  on a first step and recover  $K(s)$  on a second step from:

$$K(s) = \frac{C(s)}{1 - G_n(s)(s)C(s)} \quad (9)$$

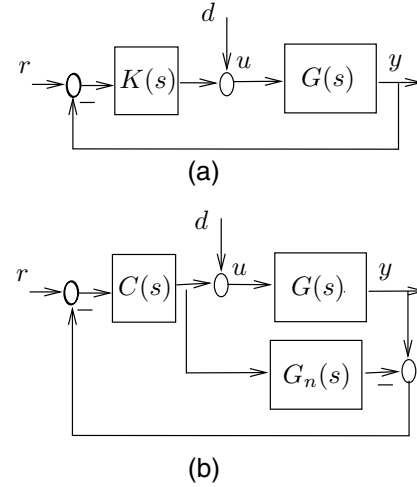


Fig. 1. Feedback Control System: (a) Conventional Configuration, (b) Internal Model Control configuration

The main feature of the IMC method is that the desired closed loop time constant is provided as a tuning parameter, commonly known as the IMC filter. Robustness is dealt through the reduction of this desired closed loop bandwidth. A detailed description of the IMC synthesis method can be found in [15] [16].

Here we will make use of the IMC formulation just to set up the min-max problem we will base the design on. This way we will directly design the  $C(s)$  transfer function as the solution to the following problem

$$\min_{C(s)} \|W(s)(M(s) - C(s)G_n(s))\|_\infty \quad (10)$$

where  $M(s)$  is a Reference Model for the closed loop system response and  $W(s)$  is a weighting function. In this communication we will use the Reference Model to specify the desired closed loop time constant,  $T_M$ . Therefore it will take the form:

$$M(s) = \frac{n_M(s)}{d_M(s)} = \frac{1}{1 + T_M s} \quad (11)$$

With respect to the weighting function,  $W(s)$ , in order to automatically include integral action and keep it as simple as possible, we will assume the following form:

$$W(s) = \frac{n_W(s)}{d_W(s)} = \gamma \frac{1 + zs}{s} \quad (12)$$

In order to include Robustness considerations, the solution to this minimization problem must be constrained to (6).

### III. SOLUTION TO THE MIN-MAX OPTIMIZATION PROBLEM

This section will present a solution to the optimization problem (10). Several approaches exist to solve this  $\mathcal{H}_\infty$  problem. See [17], [18] among others. Here we will follow a particularization of the solution presented in [19] where a polynomial approach was taken. This has the advantage of providing the structure of the optimal controller. Therefore, as we will do here, the problem statement can be constrained in order to provide a solution that leads to a PID controller.

The problem at hand is, in fact, a *min-max* approximation problem : given two transfer functions  $M(s), N(s) \in \mathcal{RH}^\infty$  find  $C(s) \in \mathcal{RH}^\infty$  such that the following cost function in the  $\infty$ -norm is minimized

$$\begin{aligned}\mathcal{J}_\infty &= \|E(s)\|_\infty \\ &= \|W(s)(M(s) - N(s)C(s))\|_\infty\end{aligned}\quad (13)$$

where  $N(s), M(s)$  and  $W(s)$  are factored as:  $N(s) = \frac{n_N(s)}{d_N(s)}$ ,  $M(s) = \frac{n_M(s)}{d_M(s)}$ ,  $W(s) = \frac{n_W(s)}{d_W(s)}$

The solution to the minimization of the cost function (13) lies in optimal interpolation theory [20]. First, factorize the plant numerator  $n_N(s)$  as  $n_N(s) = n_N^+(s)n_N^-(s)$  where the polynomial  $n_N^+(s)$  only has stable roots and  $n_N^-(s)$  is the remaining part. In order to obtain a unique factorisation the polynomial  $n_N^+(s)$  is assumed to be monic. Let  $\nu = \deg(n_N^-(s))$  and  $\{z_1, z_2, \dots, z_\nu\}$  be the distinct zeros of  $n_N^-(s)$ . From equation (13) it results that the error function  $E(s)$  is subjected to the following interpolation constraints

$$E(z_i) = M(z_i) \quad i = 1 \dots \nu \quad (14)$$

If  $z_i$  is a zero with multiplicity  $\nu$ , then additional differential interpolation constraints should be imposed.

A well established theory [20], [21], [17] that solves this problem exists and a closed form solution can be obtained from the following lemma [17]:

**Lemma 3.1:** The optimal  $E^o(s)$  which minimizes  $\|E(s)\|_\infty$  is of an all-pass form

$$E^o(s) = \begin{cases} \rho \frac{q(s)}{q^*(s)} & \text{if } \nu \geq 1 \\ 0 & \text{if } \nu = 0 \end{cases} \quad (15)$$

where  $q(s) = 1 + q_1s + q_2s^2 + \dots + q_{\nu-1}s^{\nu-1}$  is a strictly hurwitz polynomial and  $q^*(s) = q(-s)$ .

Furthermore, the constants  $\rho$  and  $\{q_i\}_{i=1}^{\nu-1}$  are real and are uniquely determined by the interpolation constraints (14). Besides, the minimum cost of (13) is given by

$$\mathcal{J}_\infty^o = \min \|E(s)\|_\infty = \|E^o(s)\|_\infty = |\rho|$$

Now we will proceed with the application of this lemma in order to compute the optimal  $C(s) = C^o(s)$ . Note first that in our case  $\nu = 1$  and  $z_1 = 1/L$ . Therefore the interpolation constraints give the following value for the optimal cost  $\rho$ :

$$\rho = |W(1/L)M(1/L)| = \gamma L \frac{(L+z)}{(L+T_M)} \quad (16)$$

Application of the above lemma gives the following equation for the optimal parameter  $C^o(s)$

$$W(s)M(s) - W(s)N(s)C^o(s) = \rho \frac{q^*(s)}{q(s)}$$

then

$$\begin{aligned}C^o(s) &= (W(s)N(s))^{-1} \left( W(s)M(s) - \rho \frac{q^*(s)}{q(s)} \right) \\ &= \frac{d_W(s)d_N(s)}{n_W(s)n_N^+(s)n_N^-(s)} \\ &\quad \left( \frac{n_W(s)n_M(s)q(s) - \rho q^*(s)d_W(s)d_M(s)}{d_W(s)d_M(s)q(s)} \right)\end{aligned}\quad (17)$$

In order for  $C^o(s)$  to be a stable transfer function,  $n_N^-(s)$  must be a factor of the numerator. That is to say, there must exist a polynomial  $\chi(s)$  such that

$$\begin{aligned}n_N^-(s)\chi(s) &= n_W(s)n_M(s)q(s) \\ &\quad - \rho q^*(s)d_W(s)d_M(s)\end{aligned}\quad (18)$$

It follows that, to determine the optimal controller  $C^o(s)$ , the  $\chi(s)$  polynomial must be known. In any case, the optimal  $C^o(s)$  will obey to the following structure:

$$C^o(s) = \frac{d_N(s)\chi(s)}{n_W(s)n_N^+(s)d_M(s)q(s)} \quad (19)$$

Moreover, as  $\nu = 1$  it follows from the previous lemma that  $q(s) = q^*(s) = 1$ . Also,  $d_N(s) = (1 + Ts)$ ,  $n_N^+(s) = K$  and  $d_M(s) = (1 + T_M(s))$ . Therefore,  $C^o(s)$  further simplifies to

$$C^o(s) = \frac{1}{K} \frac{(1 + Ts)\chi(s)}{(1 + T_Ms)(1 + zs)} \quad (20)$$

With respect to  $\chi(s)$ , it must obey to (18) so, if  $\chi(s) = \chi^0 + \chi^1s$ , then:

$$\begin{aligned}(1 - Ls)(\chi^0 + \chi^1s) &= (1 + zs) \\ &\quad - \frac{\rho s}{\gamma}(1 + T_Ms)\end{aligned}\quad (21)$$

It is easily seen that

$$\chi^0 = 1 \quad \chi^1 = z + L - \frac{\rho}{\gamma} \quad (22)$$

Therefore, the solution for the optimal  $C^o(s)$  is

$$C^o(s) = \frac{1}{K} \frac{(1 + Ts)(1 + \chi^1s)}{(1 + T_Ms)(1 + zs)} \quad (23)$$

and the resulting optimal feedback  $K^o(s) = C^o(s)/(1 - G_n(s)C^o(s))$  becomes

$$K^o(s) = \frac{1}{K(\frac{\rho}{\gamma} + T_M)} \frac{(1 + Ts)(1 + \chi^1s)}{s(1 + T_M(\frac{\rho}{\gamma} + z))} \quad (24)$$

Thus, identifying (2) and (24) the following tuning relations arise

$$\begin{aligned} K_p &= \frac{T_i}{K(\rho/\gamma + T_M)} \\ T_i &= T + \chi_1 - T_M \frac{(\rho/\gamma + z)}{(\rho/\gamma + T_M)} \\ T_d &= T_M \frac{(\rho/\gamma + z)}{(\rho/\gamma + T_M)} N \\ N &= \frac{T}{T_i} \frac{\rho/\gamma}{L} \frac{(\rho/\gamma + T_M)}{(\rho/\gamma + z)} - 1 \end{aligned} \quad (25)$$

Note that these relations provide all the PID parameters, including the derivative filter,  $N$ . The benefits of providing a tuning of this parameter have been reported in [22], [23].

Although the tuning relations (26) look somewhat complicated note they are directly expressed in terms of the process model ( $K, L$  and  $T$ ) and the definition of the optimization problem ( $\gamma, z$  and  $T_M$ ). Moreover, note that  $\gamma$  always appear as  $\rho/\gamma$ . Therefore, because of (16) it results that  $\rho/\gamma$  is independent of  $\gamma$ . Without loss of generality we can assume  $\gamma = 1$  and the previous relations simplify to:

$$\begin{aligned} K_p &= \frac{T_i}{K(\rho + T_M)} \\ T_i &= T + \chi_1 - T_M \frac{(\rho + z)}{(\rho + T_M)} \\ T_d &= T_M \frac{(\rho + z)}{(\rho + T_M)} N \\ N &= \frac{T}{T_i} \frac{\rho}{L} \frac{(\rho + T_M)}{(\rho + z)} - 1 \end{aligned} \quad (26)$$

These tuning relations provide the four ISA-PID parameters parameterized in terms of the desired  $T_M$  and  $z$  as determining the frequency range where the solution to (10) is to provide a better match. Next section gives  $z$  a meaning in terms of Robustness considerations.

#### IV. TUNING FOR ROBUSTNESS

The previous section has provided both the IMC controller structure corresponding to the optimal solution to (10) and the associated PID tuning relations (26). That problem does not take into account robustness considerations, so a further step is needed in order to deal with model uncertainty. Now we will start from the optimal solution and introduce the Robust Stability constraint.

It is worth to stress the point that our primary interest is to provide a simple way of dealing with uncertainty. From the optimal solution got in the previous section, two parameters define the optimization problem ( $T_M$  and  $z$ ). Next step will be to try to clarify the role of these parameters.

Assuming we have the set of plants defined in terms of an uncertainty description weight  $W_m(s)$ , the Robust Stability constraint takes the form:

$$\begin{aligned} &\| G_n(s)C(s)W_m(s) \|_\infty < 1 \\ \Rightarrow &\left\| K_p \frac{1 - Ls}{1 + Ts} \frac{1}{K_p} \frac{(1 + Ts)(1 + \chi^1 s)}{(1 + T_M s)(1 + zs)} W_m(s) \right\|_\infty < 1 \\ \Rightarrow &\left\| \frac{(1 - Ls)(1 + \chi^1 s)}{(1 + T_M s)(1 + zs)} W_m(s) \right\|_\infty < 1 \\ \Rightarrow &\left| \frac{(1 - Ljw)(1 + \chi^1 jw)}{(1 + T_M jw)(1 + zjw)} \right| < \left| \frac{1}{W_m(jw)} \right| \quad \forall w \end{aligned} \quad (27)$$

Now, as  $L$  is given by the process model and  $\chi^1$  by the optimization problem, the two parameters left to be chosen are  $T_M$  and  $z$ . As  $1/W_m(s)$  usually has a low pass form both parameters can contribute to provide the necessary roll-off in order to satisfy the constraint.

Moreover, we can consider  $T_M$  as a nominal specification for the closed-loop step response. Therefore, at this step it would be desirable not to change it and use  $z$  to try to meet the Robustness specification. This way, even  $z$  was defined from the weighting function of the nominal optimization problem (12), we see that it can be interpreted as a robustness parameter. The choice of  $z$  will obey to the satisfaction of a constraint of the form:

$$\left| \frac{(1 + \chi^1 jw)}{1 + zjw} \right| < \left| \frac{(1 + T_M jw)}{(1 - Ljw)} \frac{1}{W_m(jw)} \right| \quad \forall w \quad (28)$$

The choice of  $z$  becomes easy if we take into account that  $1/z$  is also a pole of the nominal closed-loop characteristic equation:

$$T(s) = \frac{(1 - Ls)(1 + \chi^1 s)}{(1 + T_M s)(1 + zs)} \quad (29)$$

This way, the optimal value for  $z$  will be the smallest one that satisfies (28). Of course, if we state a highly stringent  $T_M$  for the nominal situation, it may be possible that the value for  $z$  such that (28) is satisfied is excessively high, or even it may not exists. In such case the nominal response can deteriorate and it may be beneficial to slightly increase  $T_M$  and get lower values for  $z$ . This is the usual tradeoff between the desired nominal performance and satisfaction of the robust stability constraint. An example of application and selection of  $z$  has been presented in [24].

#### V. PROPOSED AUTOMATIC ROBUST TUNING RULE

Tuning relations (26) allows to choose  $z$  on the basis of the uncertainty levels available in each situation. In some cases, however, one may just think of having an automatic tuning. Either because there is no uncertainty description available or because of the simplicity of an automatic tuning rule where just the parameters of the FOPTD model are needed.

In order to achieve this, values for  $z$  and  $T_M$  are needed. The approach we will follow here is to propose a selection of these parameters on the basis of the Robust stability constraint (6). The usual shape for multiplicative uncertainty,

imposes  $T(s)$  a low-pass shape. This way, we will look how to assure that

$$\left| \frac{(1 + \chi^1 jw)(1 - Ljw)}{(1 + zjw)(1 + T_M jw)} \right| < 1 \quad \forall w \quad (30)$$

and to behave as a low-pass filter. The following ordering is needed:  $1/z < 1/\chi^1$  and  $1/T_M < 1/L$ . In addition, because of (22), in order to have  $1/z < 1/\chi^1$  we must have  $z > T_M$ . Note that  $z > T_M$  also implies  $1/\chi^1 < 1/T_M$ . These inequalities suggests that  $z$  and  $T_M$  could be chosen on the basis of:  $T_M = \beta L$  and  $z = \alpha T_M$  with  $\beta > 1$  and  $\alpha > 1$ . As  $\alpha$  and  $\beta$  grow the system becomes more robust.

Our choice here is to take  $\alpha = \beta = \sqrt{2}$ . This way,  $1/T_M$ , is located on the geometric center between  $1/L$  and  $1/z$ . Also, with this choice, we have  $\omega_c L \approx 0.6$ , where  $\omega_c$  is the gain crossover frequency. We have from [25] that  $\omega_c L < \pi/2 = 1.57$  provides a phase margin of at least  $45^\circ$  and  $\omega_c L < 0.7$  for  $\max_w |S(jw)| < 2$ , being  $S(s)$  the Sensitivity function. This way, the simple choice produces reasonable *classical* robustness margins and by assuring (30) is below 1 for all frequencies, we also have tolerance to mid and high frequency non modelled dynamics.

Application of the relations  $T_M = \beta L = \sqrt{2}L$  and  $z = \alpha T_M = 2L$  to tuning rule (26) provides the following simple tuning rule:

$$\begin{aligned} K_p &= \frac{T_i}{KL2.65} \\ T_i &= T + 0.03L \\ T_d &= 1.72LN \\ N &= \frac{T}{T_i} - 1 \end{aligned} \quad (31)$$

## VI. SIMULATION EXAMPLE

This section presents three examples of application of the proposed tuning rule. In order to cover possible different situations, the examples consider one lag-dominant process one lead-dominant process and one with balanced lag and delay. The examples are taken from [4]. Each one of the processes is approximated by a FOPTD model of the form (3). The three processes and corresponding FOPTD models are:

$$\begin{aligned} G^1(s) &= \frac{1}{(1+s)(1+0.1s)(1+0.01s)(1+0.001s)} \\ &\approx \frac{e^{-0.073s}}{1.073s+1} \\ G^2(s) &= \frac{e^{-s}}{(1+0.05s)^2} \approx \frac{e^{-s}}{0.093s+1} \\ G^3(s) &= \frac{1}{(1+s)^4} \approx \frac{e^{-1.42s}}{2.9s+1} \end{aligned} \quad (32)$$

In order to compare the performance of the proposed tuning rule, a comparison with respect to other three tuning rules is done:

- *S-IMC tuning rule*: This is a really simple and effective analytic tuning rule proposed in [6]. It is worth noting that for a FOPTD the S-IMC rule gives a PI controller. Therefore, we are comparing a S-IMC PI with respect to a PID.

- *AMIGO tuning rule*: Tuning rule proposed in [4] along the lines of the classical Ziegler-Nichols rules.
- *Original IMC PID*: The rule proposed in [26] for fast response is used.

With the aim of showing the step response performance preservation/degradation in the face of model variations, some degree of parametric uncertainty is assumed with respect to the parameters of the FOPTD model. This way, a variation of 30% is assumed in each one of the parameters, generating the corresponding set of plants and associated closed-loop step responses. Figures (2), (3) and (4) show the results for each one of the three scenarios, (32), considered. The step response for the original processes under the nominal situation are also shown.

It is seen that rules S-IMC and IMC-PID give the poorest results (for  $G^1(s)$  the IMC-PID even becomes unstable). On the other hand, the AMIGO tuning rules [4], as robustness criteria were taken into account implicitly on the optimization phase, show a good performance. However, specially for processes  $G^1(s)$  and  $G^3(s)$ , the proposed tuning provides a better *overall performance*. With respect to the disturbance rejection performance (not shown here due to space reasons), even not directly considered on the design phase, it is worth to say that results of the proposed and AMIGO tuning are comparable and superior to the other two Tunings. To tackle the tradeoff between solving (10) and maximize disturbance rejection is a subject of future work.

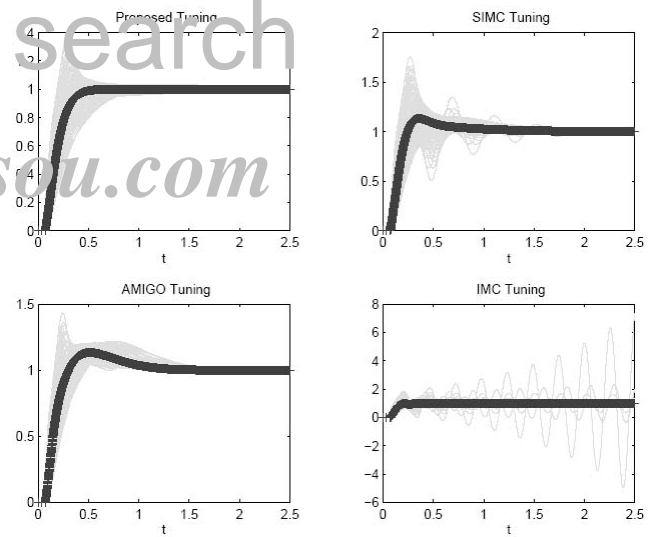


Fig. 2. Setpoint responses for process  $G^1(s)$  considering 30% variation in each one of the parameters of the FOPTD model

## VII. CONCLUSIONS

Tuning relations for PID design have been presented. In order to get results closer to Industrial applications the discussion has concentrated on the ISA formulation. The design has been done from a min-max optimization problem stated in terms of a desired time constant for the closed-loop step response. In order to cope with model uncertainty, robust stability considerations allowed to give a concise meaning

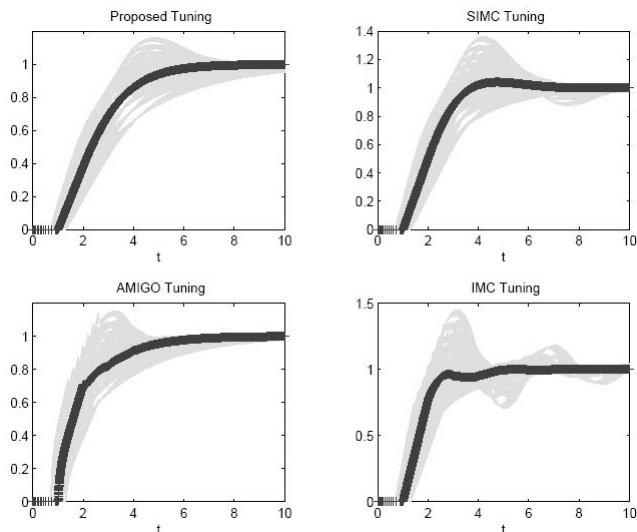


Fig. 3. Setpoint responses for process  $G^2(s)$  considering 30% variation in each one of the parameters of the FOPTD model

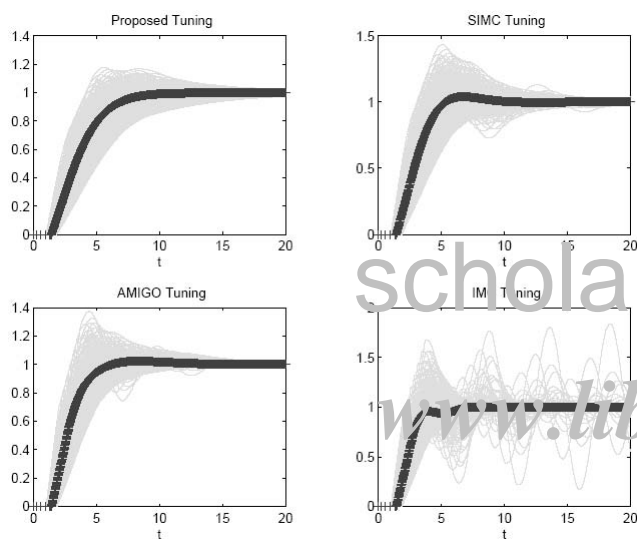


Fig. 4. Setpoint responses for process  $G^3(s)$  considering 30% variation in each one of the parameters of the FOPTD model

to the tuning parameters. As a result, the final PID tuning relations are governed by two parameters:  $T_M$  that express the desired time constant for the nominal situation, and  $z$  that allows to deal with model uncertainty. Fixed values for these parameters are also proposed and a very simple automatic tuning rule is provided. Simple examples show the performance over the whole plant family set.

Future work is conducted to introduce a Sensitivity constraint in order to improve disturbance attenuation. This way a mixed sensitivity problem will need to be solved. Although optimization approaches based on non-convex numerical methods could be used it would be helpful if an analytical solution along the lines of the one presented could be found. Also, considerations to include the set-point weights and to

design the overall ISA-PID controller are being considered.

## VIII. ACKNOWLEDGMENTS

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